On effective Kähler potential in $\mathcal{N}=2$, $d=3$ SQED

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Abstract

We compute the two-loop effective Kähler potential in three-dimensional $\mathcal{N}=2$ supersymmetric electrodynamics with Chern-Simons kinetic term for the gauge superfield. The effective action is constructed on the base of background field method with one parametric family of gauges. In such an approach, the quadratic part of quantum action mixes the gauge and matter quantum superfields yielding the complications in the computations of the loop supergraphs. To avoid this obstacle and preserve dependence on the gauge parameter we make a nonlocal change of quantum matter superfields after which the propagator is diagonalized, however the new vertices have appeared. We fix the suitable background and develop the efficient procedure of calculating the two-loop supergraphs with the new vertices. We compute the divergent and finite parts of the superfield effective action, find the two-loop effective Kähler potential and show that it does not depend on the gauge parameter.

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1 Introduction

It is well known that the leading part of the low-energy effective action in the supersymmetric field models with chiral matter superfields is described by effective Kähler potential (see, e.g., [1]). The effective Kähler potential is responsible for the structure of the quantum moduli space of $4D, \mathcal{N}=1$ gauge-matter theories in the Higgs branch and is closely related to supersymmetric sigma-models. Computations of the effective Kähler potential in the $4D, \mathcal{N}=1$ supersymmetric models have been carried out in many papers (see e.g. [2–7], [8–10] for one-loop calculations and [11] for two-loop calculations)[1].

In three-dimensional supersymmetric theories the structure of effective Kähler potential is much less well understood. The effective superpotential in $\mathcal{N}=1$ gauge-matter theories was studied in [13, 14], but it does not correspond to Kähler sigma-models for component scalar fields due to an insufficient number of supersymmetries. The two-loop effective Kähler potential was computed for the three-dimensional Wess-Zumino model in $\mathcal{N}=2$ superspace [15] but it has not been studied in gauge-matter models which have much more interesting classical and quantum properties. Note that there is a broad discussion of the structure of moduli space of three-dimensional gauge theories with $\mathcal{N}=2$ supersymmetry including its Higgs branch (see, e.g., [16–19]), but the corresponding Kähler potential has never been computed explicitly in perturbation theory. On the contrary, the low-energy effective action in the Coulomb branch of three-dimensional gauge-matter models has recently been studied in the $\mathcal{N}=2$ superspace up to two-loop order [20–24].

The aim of the present paper is to initiate the study of the perturbative quantum corrections to the effective Kähler potential in three-dimensional $\mathcal{N}=2$ supersymmetric gauge theories. We compute two-loop effective Kähler potential in three-dimensional $\mathcal{N}=2$ supersymmetric quantum electrodynamics (SQED) with Chern-Simons kinetic term for the gauge superfield. At the classical level this model is superconformal, but we show that the conformal invariance is broken by two-loop quantum corrections. We find that the two-loop Kähler potential in the $\mathcal{N}=2$ supersymmetric electrodynamics is similar in some aspects to the one-loop effective Kähler potential in four-dimensional $\mathcal{N}=1$ SQED [9].

In the present paper we study the effective Kähler potential for one particular model: $\mathcal{N}=2$ SQED with the Chern-Simons kinetic term for the gauge superfield and two chiral superfields. The following are arguments supporting the study the effective Kähler potential in this model:

- Although this model is quite simple, it possesses a non-trivial effective Kähler potential which represents the leading part of the low-energy effective action in the Higgs branch. Note that, in contrast to the four-dimensional case, we need to study the two-loop effective action since, as we will show further, the one-loop quantum corrections to the effective Kähler potential are trivial in the sense that they repeat the form of classical Kähler potential. In general, computation of two-loop quantum corrections is a hard routine, but in the present case we need to consider just a few two-loop Feynman graphs since the model is Abelian and, in particular, ghost superfields do not contribute.

- As we will show further, the form of two-loop quantum corrections to the effective Kähler potential is in fact dictated by logarithmically divergent supergraphs. Hence, the effective

[1] The detailed analysis of the $4D$ superfield effective potentials has been given in the thesis [12].

Kähler potential which is proper to $\mathcal{N} = 2$ Chern-Simons-matter theories it seems can not appear in three-dimensional models such as $\mathcal{N} > 2$ Chern-Simons-matter gauge theories which have no UV divergences [25–27] or the $\mathcal{N} = 2$ SQED with Maxwell kinetic term for the gauge superfield which is superrenormalizable.

- The $\mathcal{N} = 2$ SQED with the Chern-Simons kinetic term is classically superconformal, but, as we will show, the two-loop quantum corrections to the low-energy effective action break the conformal invariance. This is analogous to the holomorphic low-energy effective action in four-dimensional $\mathcal{N} = 2$ gauge theories [28] which is known to be responsible for the superconformal symmetry breaking.

- We consider the $\mathcal{N} = 2$ SQED with two chiral superfields having different charges with respect to the gauge superfield. This model is advantageous as compared to similar models with odd number of chiral superfields which may have parity anomaly [29–31]. Moreover, in the considered model the effective Kähler potential can be unambiguously computed within the background field method since we can fix the background for chiral matter superfields which solves classical equations of motion. As a result, the obtained effective Kähler potential corresponds to the gauge-independent part of the effective action.

Let us discuss several technical points concerning two-loop computations in the considered model. The effective action in quantum field theory of gauge fields is known to be a gauge-dependent quantity. However, the effective action calculated for background field satisfying the effective equations of motions is gauge independent (see e.g. [32]). When we study the perturbative quantum corrections to effective action in the frame of loop expansion, the gauge independent one-loop corrections should be considered on the classical equations of motion while for the gauge independent two-loop quantum corrections we have to take into account the effective equations of motion up to one-loop order. In the $\mathcal{N} = 2$ SQED studied in the present paper it is sufficient to consider constant background chiral superfields to compute the effective Kähler potential. As we will demonstrate, this background obeys not only classical but also quantum effective equations of motion up to one-loop order. This guarantees that the two-loop effective Kähler potential computed in this model is gauge independent. Moreover, in the functional integral we fix the gauge freedom, but we keep the gauge-fixing parameter α arbitrary throughout all quantum computations. We directly demonstrate that the obtained one- and two-loop quantum corrections to the effective Kähler potential are independent of α, confirming its gauge independence.

Another technical comment concerns the details of applications of the background field method at the two-loop order. When we perform the background quantum splitting the classical action acquires a number of terms which mix gauge and matter superfields at the quadratic order and make the propagator non-diagonal. In quantum computations it is desirable to deal with the diagonal propagator for quantum superfields. Otherwise the computations become extremely complicated. There are, in general, two ways to achieve this: (i) to make a non-local change of fields to diagonalize the propagator or (ii) to apply a generalized gauge-fixing condition (R_ξ-gauge) which eliminates the mixed terms at the quadratic order. The latter approach is usually simpler, but it does not allow one to keep the gauge-fixing parameter arbitrary. Therefore, in the present work we make a non-local change of quantum superfields to bring the propagator to the diagonal form. The cost for this is that we get new interaction

vertices having non-local form and playing important role in two-loop quantum computations. This means we should develop a specific technique to compute the supergraphs with the new vertices.

The rest of this paper is organized as follows. Section 2 is devoted to some preliminary discussion concerning the structure of loop quantum corrections to the effective Kähler potential and specify the background which is suitable for its evaluation. In section 3 we perform the background-quantum splitting and derive the form of propagators and interaction vertices which will be employed in loop quantum computations. In the next two sections we calculate one- and two-loop quantum effective actions and derive the form of effective Kähler potential at the two-loop order. In the last section we discuss the possible extensions of the results of the present paper. In the appendices we collect some technical details of two-loop quantum computations. Throughout this paper we use the $\mathcal{N}=2$, $d=3$ superspace notations and conventions introduced in earlier works [20–24].

2 Classical action and specification of background

We consider the three-dimensional $\mathcal{N}=2$ supersymmetric electrodynamics which is described by two chiral matter superfields Q_+ and Q_- and a gauge superfield V with superfield strength $G=\frac{i}{2}\bar{D}^\alpha D_\alpha V$. In general at a classical level such a model can have several parameters: the complex and real mass parameters of the chiral superfields, the topological mass of the gauge superfield and the Fayet-Iliopoulos term. In the present paper we consider a particular case where the masses of chiral superfields are vanishing and the gauge superfield has infinite topological mass. Moreover, when the Fayet-Iliopoulos term vanishes, the model is superconformal at the classical level. The only parameter in the classical action is the Chern-Simons level k

$$S=\frac{k}{2\pi}\int d^7z\,VG-\int d^7z\,(\bar{Q}_+e^{2V}Q_++\bar{Q}_-e^{-2V}Q_-)\,. \tag{2.1}$$

The corresponding classical equations of motion are

$$G=\frac{2\pi}{k}(\bar{Q}_+e^{2V}Q_+-\bar{Q}_-e^{-2V}Q_-)\,, \tag{2.2a}$$
$$\bar{D}^2(\bar{Q}_\pm e^{\pm 2V})=0\,,\qquad D^2(Q_\pm e^{\pm 2V})=0\,. \tag{2.2b}$$

The most natural approach to study the quantum effective action is the background field method. For gauge theories in the $\mathcal{N}=1$ $d=4$ superspace this method is discussed in [33]. Basic features of this method for $\mathcal{N}=2$ $d=3$ superspace were formulated in [20–22]. For recent applications of this method for computing the effective actions in three-dimensional gauge theories in the sector of gauge superfield see [23, 24]. Following this method, we split the original superfields V, $Q_\pm$ and $\bar{Q}_\pm$ into the so-called 'quantum' v, $q_\pm$, $\bar{q}_\pm$ and 'background' V, $Q_\pm$, $\bar{Q}_\pm$ parts[2]

$$V\to v+V\,,\quad Q_\pm\to q_\pm+Q_\pm\,,\quad \bar{Q}_\pm\to\bar{q}_\pm+\bar{Q}_\pm\,. \tag{2.3}$$

[2]Note that we denote the background superfields by the same letters as the original ones. We hope that this will not lead to misunderstandings since after the background-quantum splitting the original superfields never appear.

Then, after fixing the gauge freedom for the quantum gauge superfield v, one obtains a gauge-invariant effective action for the background superfields

$$\Gamma = \Gamma[V, Q_\pm, \bar{Q}_\pm]\,. \tag{2.4}$$

In the resent paper, we restrict ourselves to considering only the part of the effective action which is described by the effective Kähler potential for the chiral superfields $K_{\rm eff}(Q_\pm, \bar{Q}_\pm)$. For this purpose it is sufficient to consider the background superfields subject to the following constraints

$$V = 0\,, \quad D_\alpha Q_\pm = 0\,, \quad \bar{D}_\alpha \bar{Q}_\pm = 0\,. \tag{2.5}$$

In general, the effective action is gauge dependent but, if the background superfields satisfy the exact effective equations of motions, the effective action is gauge independent (see e.g. [32]). To get the gauge independent one-loop effective action it is sufficient to use the background superfields obeying the classical equations of motion. For the two-loop effective action we should take the background fields satisfying the one-loop equations of motion. In the case under consideration we assume that the background superfields obey not only (2.5), but also (2.2),

$$\bar{Q}_+ Q_+ = \bar{Q}_- Q_- \equiv \bar{Q}Q\,. \tag{2.6}$$

In principle, this constraint could be modified by one-loop corrections, but as we will show further, this is not the case and it can be safely used for two-loop computations as well. Thus, the problem is reduced to finding the effective action which is described by a single function $K_{\rm eff}(\bar{Q}Q)$

$$\Gamma = -\int d^7 z\, K_{\rm eff}(\bar{Q}Q)\,. \tag{2.7}$$

We emphasize that this part of the effective action is gauge independent and can be unambiguously computed in the two-loop approximation for the background superfields constrained by (2.5) and (2.6).

In the present paper we will compute the effective Kähler potential up to two-loop order in the quantum perturbation theory

$$K_{\rm eff} = K + K^{(1)} + K^{(2)} + \ldots \tag{2.8}$$

Here K is the classical Kähler potential, $K = 2\bar{Q}Q$ while $K^{(1)}$ and $K^{(2)}$ correspond to one- and two-loop quantum contributions. The ellipsis stand for higher loop quantum corrections which are beyond our considerations.

3 Propagators and vertices

Upon the background-quantum splitting (2.3), the classical action (2.1) is expanded in a series over the quantum superfields,

$$S = S_0 + S_1 + S_2 + S_{\rm int} + \ldots\,, \tag{3.1}$$

where S_0 is the action depending only on the background superfields and having the form (2.1); S_1 is liner in quantum superfields part and does not give rise to one-particle irreducible diagrams for effective action. The action S_2 is quadratic in quantum superfields

$$
\begin{aligned}
S_2 &= \int d^7z\, v\left(\frac{ik}{4\pi}\bar{D}^\alpha D_\alpha + M\right)v - \int d^7z\,(\bar{q}_+q_+ + \bar{q}_-q_-) \\
&\quad - 2\int d^7z\,(\bar{Q}_+\,q_+v + Q_+\,\bar{q}_+v - \bar{Q}_-\,q_-v - Q_-\,\bar{q}_-v)\,,
\end{aligned}
\tag{3.2}
$$

while $S_{\rm int}$ is responsible for cubic and quartic vertices for quantum superfields,

$$
\begin{aligned}
S_{\rm int} &= -\int d^7z\Big(2\bar{Q}_+\,q_+v^2 + 2Q_+\,\bar{q}_+v^2 + 2\bar{Q}_-\,q_-v^2 + 2Q_-\,\bar{q}_-v^2 \\
&\quad + 2\bar{q}_+q_+v - 2\bar{q}_-q_-v + 2\bar{q}_+q_+v^2 + 2\bar{q}_-q_-v^2 - \frac{1}{3}Mv^4 \\
&\quad + \frac{4}{3}\bar{q}_+v^3 + \frac{4}{3}q_+v^3 - \frac{4}{3}\bar{q}_-v^3 - \frac{4}{3}\bar{q}_-v^3 - \frac{4}{3}(\bar{Q}_+Q_+ - \bar{Q}_-Q_-)v^3\Big)\,,
\end{aligned}
\tag{3.3}
$$

where

$$
M \equiv -2(\bar{Q}_+Q_+ + \bar{Q}_-Q_-) = -4\,\bar{Q}Q\,. \tag{3.4}
$$

The ellipses in (3.1) stand for higher order interaction vertices for quantum superfields which are irrelevant for two-loop computations.

The operator $\bar{D}^\alpha D_\alpha$ in (3.2) is degenerate and requires gauge fixing. We use standard gauge fixing conditions in the $\mathcal{N}=2$ $d=3$ superspace

$$
\bar{D}^2 v = 0\,, \qquad D^2 v = 0\,, \tag{3.5}
$$

which can be effectively taken into account by adding to (3.2) the following gauge-fixing action [25, 26, 34]

$$
S_{\rm gf} = \frac{ik\alpha}{8\pi}\int d^7z\, v(D^2 + \bar{D}^2)v\,, \tag{3.6}
$$

where α is a real parameter. In the present paper we do not fix this parameter and keep it arbitrary. As we will show further, the effective Kähler potential is independent of this parameter. It means in fact that we study the gauge-independent part of the effective action.

We consider the Abelian gauge theory, therefore the ghost superfields, associated with the gauge fixing (3.5), decouple and do not contribute to the effective action.

Note that the quadratic action S_2 contains the mixed gauge and matter quantum field terms given in the second line of (3.2). This unpleasant feature leads to non-diagonal propagator for the quantum superfields and makes quantum loop computations more involved. However, it is always possible to make a non-local change of (anti)chiral superfields in the functional integral

such that the propagator becomes diagonal. For the action (3.2) such a change of fields reads [3]

$$
\begin{aligned}
q_+(z) &\to q_+(z) + 2\int d^7z' G_{+-}(z,z')Q_+(z')v(z')\,,\\
q_-(z) &\to q_-(z) - 2\int d^7z' G_{+-}(z,z')Q_-(z')v(z')\,,\\
\bar q_+(z) &\to \bar q_+(z) + 2\int d^7z' G_{-+}(z,z')\bar Q_+(z')v(z')\,,\\
\bar q_-(z) &\to \bar q_-(z) - 2\int d^7z' G_{-+}(z,z')\bar Q_-(z')v(z')\,,
\end{aligned}
\tag{3.7}
$$

where G_{+-} and G_{-+} are propagators for the (anti)chiral superfields obeying the equations

$$
\frac14 D^2 G_{+-}(z,z') = -\delta_-(z,z')\,,\qquad \frac14 \bar D^2 G_{-+}(z,z') = -\delta_+(z,z')\,. \tag{3.8}
$$

Here $\delta_\pm(z,z')$ are chiral and anti-chiral delta-functions which are related to the full superspace delta-function $\delta^7(z-z')$ as

$$
\delta_+(z,z') = -\frac14 \bar D^2\delta^7(z-z')\,,\qquad \delta_-(z,z') = -\frac14 D^2\delta^7(z-z')\,. \tag{3.9}
$$

The propagators G_{+-} and G_{-+} obeying (3.8) have the following explicit form

$$
G_{+-}(z,z') = \frac{1}{\Box}\frac{\bar D^2 D^2}{16}\delta^7(z-z')\,,\qquad G_{-+}(z,z') = \frac{1}{\Box}\frac{D^2 \bar D^2}{16}\delta^7(z-z')\,. \tag{3.10}
$$

Indeed, after the change of fields (3.7) the action (3.2) acquires the form

$$
\begin{aligned}
S_2 + S_{\rm gf} &= \int d^7z\, v\left(\frac{ik}{4\pi}\bar D^\alpha D_\alpha + \frac{ik\alpha}{8\pi}(\bar D^2 + D^2) + M\right)v - \int d^7z\,(\bar q_+ q_+ + \bar q_- q_-)\\
&\quad - \int d^7z\, d^7z'\, v(z)v(z')M\frac{\{\bar D^2, D^2\}}{16\Box}\delta^7(z-z')\,.
\end{aligned}
\tag{3.11}
$$

Here, in the last term, we used the fact that the spinor derivatives do not act on the background superfields according to (2.5).

It is important to note that gauge superfield v remains unchanged when we change the chiral superfields as in (3.7). Thus, though these transformations have the non-local form the Jacobian of this change of fields is equal to unit.

The action (3.11) shows that the chiral superfields have conventional free propagators,

$$
i\langle q_+(z)\bar q_+(z')\rangle = i\langle q_-(z)\bar q_-(z')\rangle = G_{+-}(z,z')\,, \tag{3.12}
$$

while the Green's function for the quantum v-superfield obeys

$$
(H + M - \Delta)G(z,z') = -\delta^7(z-z')\,, \tag{3.13}
$$

[3] A similar non-local change of fields was used in [35] within computations of one-loop effective Kähler potential in four-dimensional $\mathcal{N}=1$ SQED (see also earlier paper [36] for non-local change of fields in non-supersymmetric QED).

where we introduced the notations

$$
\begin{aligned}
H &= \frac{ik}{4\pi}\left(\bar{D}^\alpha D_\alpha + \frac{\alpha}{2}(\bar{D}^2 + D^2)\right), && (3.14)\\
\Delta &= M\frac{\{\bar{D}^2, D^2\}}{16\Box}. && (3.15)
\end{aligned}
$$

In Appendix A we demonstrate that the solution of (3.13) can be represented in the form

$$
G(z,z') = \left[\frac{i\pi}{k}\frac{\bar{D}^\alpha D_\alpha}{\Box + \frac{4\pi^2 M^2}{k^2}} - \frac{\pi^2 M}{k^2}\frac{(\bar{D}^\alpha D_\alpha)^2}{\Box(\Box + \frac{4\pi^2 M^2}{k^2})} + \frac{i\pi}{2k\alpha}\frac{\bar{D}^2 + D^2}{\Box}\right]\delta^7(z-z'). \tag{3.16}
$$

As we have just shown, the change of fields (3.7) eliminates the mixed propagators among the gauge and matter superfields. The price for this is a complication of the part of the action responsible for the interaction vertices. Indeed, after the change of fields (3.7) the action (3.3) acquires many new vertices which have non-local form

$$
\begin{aligned}
S_{\rm int} = &- \int d^7z\Big(2\bar{Q}\,q_+v^2 + 2Q\,\bar{q}_+v^2 + 2\bar{Q}\,q_-v^2 + 2Q\,\bar{q}_-v^2 \\
&\quad + 2\bar{q}_+q_+v - 2\bar{q}_-q_-v + 2\bar{q}_+q_+v^2 + 2\bar{q}_-q_-v^2 + \frac{4}{3}\bar{Q}Q\,v^4\Big) \\
&+ \int d^7z\,d^7z'\Big(4\bar{Q}_+G_{-+}(z,z')\,q_+(z)\,v(z)\,v(z') + 4Q_+G_{+-}(z,z')\,\bar{q}_+(z)\,v(z)\,v(z') \\
&\quad + 4\bar{Q}_-G_{-+}(z,z')\,q_-(z)\,v(z)\,v(z') + 4Q_-G_{+-}(z,z')\,\bar{q}_-(z)\,v(z)\,v(z')\Big) \\
&+ 16\int d^7z\,d^7z'\,d^7z''\,\bar{Q}QG_{+-}(z,z')\,G_{-+}(z,z'')\,v^2(z)\,v(z')\,v(z'') + \ldots
\end{aligned} \tag{3.17}
$$

Here dots stand for several more terms which have the structure $q_\pm v^3$ and $\bar{q}_\pm v^3$. We omit these terms as the corresponding vertices cannot appear in the two-loop Feynman diagrams since we have no mixed $\langle qv\rangle$ and $\langle\bar{q}v\rangle$ propagators. We emphasize that the expression (3.17) is a result of identical transformation in local field theory.

4 One-loop effective Kähler potential

The action (3.11) specifies the one-loop quantum corrections to the effective action. The (anti)chiral superfields are free and do not contribute. Thus, the one-loop effective action is given by the trace of the logarithm of the operator of quadratic fluctuations for the superfield v

$$
\Gamma^{(1)} = \frac{i}{2}\mathrm{Tr}\ln(H + M - \Delta), \tag{4.1}
$$

where M is the effective mass given by (3.4) and the operators H and Δ are defined in (3.14) and (3.15). It is convenient to represent (4.1) as a sum of two terms

$$
\Gamma^{(1)} = \frac{i}{2}\mathrm{Tr}\ln(H + M) + \frac{i}{2}\mathrm{Tr}\ln(1 - (H+M)^{-1}\Delta), \tag{4.2}
$$

and compute them separately.

In the first term in (4.2) we expand the logarithm in a series

$$\frac{i}{2}\mathrm{Tr}\ln(H+M) \;=\; \frac{i}{2}\int d^7z\,d^7z'\,\delta^7(z-z')\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{nM^n}H^n\delta^7(z'-z) \tag{4.3}$$

and for the terms in this series we apply the following identities

$$(\bar{D}^\alpha D_\alpha)^n \;=\; \begin{cases} (4\Box)^{k-1}\bar{D}^\alpha D_\alpha & n=2k-1 \\ (4\Box)^{k-1}(\bar{D}^\alpha D_\alpha)^2 & n=2k \end{cases}, \tag{4.4a}$$

$$(\bar{D}^2+D^2)^n \;=\; \begin{cases} (16\Box)^{k-1}(\bar{D}^2+D^2) & n=2k-1 \\ (16\Box)^{k-1}\{\bar{D}^2,D^2\} & n=2k \end{cases}. \tag{4.4b}$$

Only the terms with four covariant spinor derivatives give non-trivial rises owing to the standard identity

$$\delta^4(\theta-\theta')\frac{1}{16}D^2\bar{D}^2\delta^7(z-z') = \delta^7(z-z')\,. \tag{4.5}$$

Then, the expression (4.3) gets local form in the Grassmann variables

$$\begin{aligned}\frac{i}{2}\mathrm{Tr}\ln(H+M) \;=\; & -\frac{i}{2}\int d^3xd^3x'd^4\theta\,\delta^3(x-x')\frac{1}{\Box}\ln\left(\Box+\frac{4\pi^2M^2}{k^2}\right)\delta^3(x-x') \\ & +\frac{i}{2}\int d^3xd^3x'd^4\theta\,\delta^3(x-x')\frac{1}{\Box}\ln\left(\Box+\frac{4\pi^2M^2}{\alpha^2k^2}\right)\delta^3(x-x')\,.\end{aligned} \tag{4.6}$$

Next, we make the Fourier transform for the delta-functions and compute the momentum integrals

$$\int\frac{d^3p}{(2\pi)^3}\frac{1}{p^2}\ln\left(1-\frac{m^2}{p^2}\right) = -\frac{i}{2\pi}|m|\,, \tag{4.7}$$

and get the following result for the term (4.3)

$$\frac{i}{2}\mathrm{Tr}\ln(H+M) = \frac{2}{k}\left(1-\frac{1}{\alpha}\right)\int d^7z\,\bar{Q}Q\,. \tag{4.8}$$

To evaluate the second term in (4.2) we introduce the Green's function $\mathcal{G}(z,z')$ of the operator $H+M$ in (3.14)

$$(H+M)\mathcal{G}(z,z') \;=\; -\delta^7(z-z')\,, \tag{4.9}$$

$$\begin{aligned}\mathcal{G}(z,z') \;=\; & \left[\frac{i\pi}{k}\frac{\bar{D}^\alpha D_\alpha}{\Box+\frac{4\pi^2M^2}{k^2}}+\frac{i\pi}{2k\alpha}\frac{\bar{D}^2+D^2}{\Box+\frac{4\pi^2M^2}{k^2\alpha^2}}\right. \\ & \left.+\frac{\pi^2M}{2k^2}\frac{D^\alpha\bar{D}^2D_\alpha}{\Box(\Box+\frac{4\pi^2M^2}{k^2})}-\frac{\pi^2M}{4k^2\alpha^2}\frac{\{\bar{D}^2,D^2\}}{\Box(\Box+\frac{4\pi^2M^2}{k^2\alpha^2})}\right]\delta^7(z-z')\,.\end{aligned} \tag{4.10}$$

This allows us to represent the second term in (4.2) as

$$\begin{aligned}\frac{i}{2}\mathrm{Tr}\ln(1-(H+M)^{-1}\Delta) \;=\; & \frac{i}{2}\mathrm{Tr}\ln(1+\mathcal{G}\Delta) \\ =\; & \frac{i}{2}\mathrm{Tr}\ln\left(1+A\frac{\bar{D}^2+D^2}{4}+B\frac{\{\bar{D}^2,D^2\}}{16}\right),\end{aligned} \tag{4.11}$$

where

$$A = \frac{2\pi i M}{k\alpha}\frac{1}{(\Box + \frac{4\pi^2 M^2}{k^2\alpha^2})}\,, \qquad B = -\frac{4\pi^2 M^2}{k^2\alpha^2}\frac{1}{\Box(\Box + \frac{4\pi^2 M^2}{k^2\alpha^2})}\,. \tag{4.12}$$

The argument of the log function in (4.11) can be represented as a product of two factors

$$1 + A\frac{\bar D^2 + D^2}{16} + B\frac{\{\bar D^2, D^2\}}{16} \quad = \quad \left(1 + \frac{A}{1+\Box B}\frac{\bar D^2 + D^2}{4}\right)\left(1 + B\frac{\{\bar D^2, D^2\}}{16}\right). \tag{4.13}$$

Hence, we have the sum of two terms

$$\frac{i}{2}\mathrm{Tr}\ln(1-(H+M)^{-1}\Delta) = \frac{i}{2}\mathrm{Tr}\ln\left(1+\frac{A}{1+\Box B}\frac{\bar D^2 + D^2}{4}\right) + \frac{i}{2}\mathrm{Tr}\ln\left(1+B\frac{\{\bar D^2, D^2\}}{16}\right). \tag{4.14}$$

We expand these log functions in series and in each term apply the identities (4.4). As a result, we get

$$\frac{i}{2}\mathrm{Tr}\ln(1-(H+M)^{-1}\Delta) = \frac{i}{2}\int d^3x d^3x' d^4\theta\, \delta^3(x-x')\frac{1}{\Box}\ln\left(\frac{\Box}{\Box + \frac{4\pi^2 M^2}{\alpha^2 k^2}}\right)\delta^3(x-x')\,. \tag{4.15}$$

This expression leads to the same momentum integral (4.7). Hence, we conclude

$$\frac{i}{2}\mathrm{Tr}\ln(1-(H+M)^{-1}\Delta) = \frac{2}{k\alpha}\int d^7z\,\bar Q Q\,. \tag{4.16}$$

As a result, the one-loop effective action is given by the sum of (4.8) and (4.16)

$$\Gamma^{(1)} \quad = \quad \int d^7z\, K^{(1)}\,, \tag{4.17a}$$

$$K^{(1)} \quad = \quad -\frac{2}{k}\,\bar Q Q\,. \tag{4.17b}$$

As expected, the one-loop effective Kähler potential (4.17b) does not contain ultraviolet divergences and is independent of the gauge-fixing parameter α.

5 Two-loop effective action

It is well known that the two-loop quantum contributions to the effective action are usually represented by the Feynman graphs having two different topologies which we call "Θ" and "∞". These diagrams involve cubic and quartic vertices originating from the action (3.17). The lines in these diagrams correspond to either chiral or gauge superfield propagators given by (3.10) and (3.16), respectively. In this section we compute those two-loop Feynman graphs which contribute to the effective Kähler potential, starting with the diagrams of topology "∞".

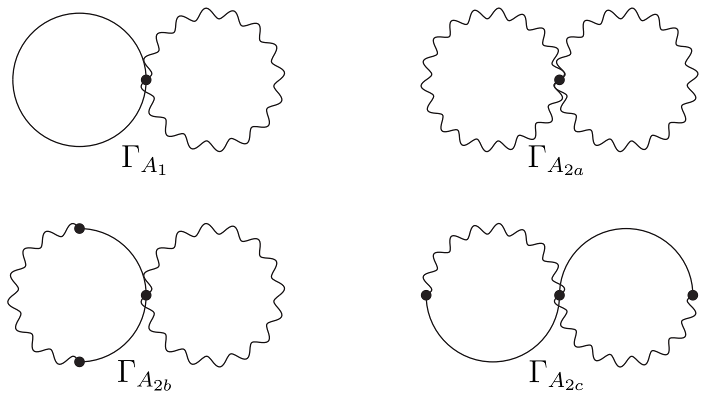

Figure 1: Two-loop Feynman graphs with quartic vertices.

5.1 Graphs with quartic vertices

The action (3.17) contains two types of quartic vertices: one vertex of v^4 type and the other one of $\bar{q}qv^2$ type. Correspondingly, there are two types of two-loop quantum contributions to the effective action with these vertices

$$\Gamma_{A_1} = -\int d^5z_1\, d^5\bar{z}_2\, d^7z_3\, d^7z_4\, \frac{\delta^4 S}{\delta v(z_4)\delta v(z_3)\delta\bar{q}(z_2)\delta q(z_1)}\, G_{+-}(z_1,z_2)\, G(z_3,z_4)\,, \tag{5.1}$$

$$\Gamma_{A_2} = -\frac{1}{8}\int d^7z_1\, d^7z_2\, d^7z_3\, d^7z_4\, \frac{\delta^4 S}{\delta v(z_4)\delta v(z_3)\delta v(z_2)\delta v(z_1)} G(z_1,z_2)\, G(z_3,z_4)\,. \tag{5.2}$$

Note that the vertex in (5.1) has simple local form

$$\frac{\delta^4 S}{\delta v(z_4)\delta v(z_3)\delta\bar{q}(z_2)\delta q(z_1)} = -4\Big(-\frac{\bar{D}_1^2}{4}\Big)\Big(\delta_-(z_1-z_2)\delta^7(z_1-z_3)\delta^7(z_1-z_4)\Big)\,. \tag{5.3}$$

Using this expression we restore the full superspace measure in (5.1), $d^7z_1 = -\frac{1}{4}\bar{D}_1^2 d^5z_1$ and integrate over z_2, z_3 and z_4 using the delta-functions

$$\Gamma_{A_1} = 4\int d^7z\, G_{+-}(z,z)\, G(z,z)\,. \tag{5.4}$$

This contribution can be visualized by the Feynman graph Γ_{A_1} in Fig. 1. Formally, the propagators (3.10) and (3.16) have enough D-factors to get a non-trivial result, but the bosonic part of the propagator G_{+-} vanishes at coincident space-time points in the frame of dimensional regularization. Thus, this Feynman graph does not contribute to the effective Kähler potential

$$\Gamma_{A_1} = 0\,. \tag{5.5}$$

We point out that, in general, this Feynman graph is non-trivial, but it vanishes on the background (2.5).

Consider now the contribution (5.2). It contains the quartic vertex which has the following form

$$
\begin{aligned}
\frac{\delta^4 S}{\delta v(z_4)\delta v(z_3)\delta v(z_2)\delta v(z_1)} &= -32\,\bar{Q}Q\,\delta^7(z_1-z_2)\delta^7(z_1-z_3)\delta^7(z_1-z_4) \\
-32\bar{Q}Q\Big(& G_{+-}(z_2,z_3)G_{-+}(z_2,z_1)\delta^7(z_2-z_4) + G_{+-}(z_2,z_4)G_{-+}(z_2,z_1)\delta^7(z_2-z_3) \\
&+ G_{+-}(z_3,z_2)G_{-+}(z_3,z_1)\delta^7(z_3-z_4) + G_{+-}(z_3,z_1)G_{-+}(z_3,z_2)\delta^7(z_3-z_4) \\
&+ G_{+-}(z_2,z_1)G_{-+}(z_2,z_3)\delta^7(z_2-z_4) + G_{+-}(z_2,z_1)G_{-+}(z_2,z_4)\delta^7(z_2-z_3) \\
&+ G_{+-}(z_1,z_2)G_{-+}(z_1,z_3)\delta^7(z_1-z_4) + G_{+-}(z_1,z_2)G_{-+}(z_1,z_4)\delta^7(z_1-z_3) \\
&+ G_{+-}(z_1,z_3)G_{-+}(z_1,z_2)\delta^7(z_1-z_4) + G_{+-}(z_1,z_4)G_{-+}(z_1,z_2)\delta^7(z_1-z_3) \\
&+ G_{+-}(z_1,z_4)G_{-+}(z_1,z_3)\delta^7(z_1-z_2) + G_{+-}(z_1,z_3)G_{-+}(z_1,z_4)\delta^7(z_1-z_2)\Big)\,. \qquad (5.6)
\end{aligned}
$$

The term in the first line corresponds to the local part of this vertex while the other terms are non-local since they involve the Green's functions G_{+-} and G_{-+}. The contribution from the local part of this vertex is represented by the Feynman graph $\Gamma_{A_{2a}}$ in Fig. 1. The other non-local terms in (5.6) correspond to $\Gamma_{A_{2b}}$ and $\Gamma_{A_{2c}}$.

Let us consider first the Feynman graph $\Gamma_{A_{2a}}$ corresponding to the local part of the vertex (5.6). Using the delta-functions in this vertex we integrate over dz_2, dz_3 and dz_4

$$
\Gamma_{A_{2a}} = 4\int d^7z\,\bar{Q}Q\,G(z,z)\,G(z,z)\,. \qquad (5.7)
$$

To compute this expression we have to consider the Green's function of the gauge superfield (3.16) at coincident superspace points. The details of these computations are collected in Appendix B.1. The result is

$$
\Gamma_{A_{2a}} = -\frac{4}{k^2}\int d^7z\,\bar{Q}Q\,. \qquad (5.8)
$$

The Feynman graphs $\Gamma_{A_{2b}}$ and $\Gamma_{A_{2c}}$ in Fig. 1 correspond to the following analytic expressions

$$
\begin{aligned}
\Gamma_{A_{2b}} &= 16\int d^7z_1 d^7z_2 d^7z_3\,\bar{Q}Q\,G_{+-}(z_1,z_3)G_{-+}(z_2,z_3)G(z_1,z_2)\,G(z_3,z_3) = 0\,, \qquad (5.9) \\
\Gamma_{A_{2c}} &= 16\int d^7z_1 d^7z_2 d^7z_3\,\bar{Q}Q\,G_{+-}(z_3,z_2)G_{-+}(z_1,z_2)\,G(z_1,z_2)G(z_3,z_2) = 0\,. \qquad (5.10)
\end{aligned}
$$

To evaluate these expressions we have to integrate by parts the covariant spinor derivatives which are present in the propagators G_{+-} given in (3.10). It is possible to distribute the derivatives in such a way that the operator $\bar{D}^2D^2$ acts on the propagator (3.16). Then, it is easy to see that both contributions (5.9), (5.10) vanish owing to the identity (A.9)

$$
\Gamma_{A_{2b}} = \Gamma_{A_{2c}} = 0\,. \qquad (5.11)
$$

We conclude that the Feynman graphs represented in Fig. 1 give rise to the effective action (5.8). The corresponding part of the effective Kähler potential has a form similar to (4.17b).

5.2 Graphs with cubic vertices

According to the action (3.17), there are the following three types of cubic vertices

$$\frac{\delta^3 S_{\rm int}}{\delta v(z_3)\delta\bar q_\pm(z_2)\delta q_\pm(z_1)} = \pm(-2)\Big(-\frac{\bar D_1^2}{4}\Big)\Big(\delta_-(z_1-z_2)\delta^7(z_1-z_3)\Big), \tag{5.12}$$

$$\begin{aligned}\frac{\delta^3 S_{\rm int}}{\delta v(z_3)\delta v(z_2)\delta q_\pm(z_1)} &= -4\bar Q_\pm\Big(-\frac{\bar D_1^2}{4}\Big)\Big(\delta^7(z_1-z_2)\delta^7(z_1-z_3)\Big) \\ &+4\bar Q_\pm\Big(-\frac{\bar D_1^2}{4}\Big)\Big(G_{-+}(z_1,z_2)\delta^7(z_1-z_3)+G_{-+}(z_1,z_3)\delta^7(z_1-z_2)\Big),\end{aligned} \tag{5.13}$$

$$\begin{aligned}\frac{\delta^3 S_{\rm int}}{\delta v(z_3)\delta v(z_2)\delta\bar q_\pm(z_1)} &= -4Q_\pm\Big(-\frac{D_1^2}{4}\Big)\Big(\delta^7(z_1-z_2)\delta^7(z_1-z_3)\Big) \\ &+4Q_\pm\Big(-\frac{D_1^2}{4}\Big)\Big(G_{+-}(z_1,z_2)\delta^7(z_1-z_3)+G_{+-}(z_1,z_3)\delta^7(z_1-z_2)\Big).\end{aligned} \tag{5.14}$$

Using these vertices and the propagators (3.10), (3.16) it is possible to construct the following two types of two-loop contributions to the effective action

$$\begin{aligned}\Gamma_{B_1} &= -\int d^5z_1\,d^5\bar z_2\,d^7z_3\,d^5z_4\,d^5\bar z_5\,d^7z_6\,\frac{\delta^3 S}{\delta v(z_3)\delta\bar q(z_2)\delta q(z_1)}\cdot\frac{\delta^3 S}{\delta v(z_6)\delta\bar q(z_5)\delta q(z_6)} \\ &\quad\times G_{+-}(z_1,z_5)G_{+-}(z_4,z_2)G(z_3,z_6)\,,\end{aligned} \tag{5.15}$$

$$\begin{aligned}\Gamma_{B_2} &= -\frac12\int d^5z_1\,d^7z_2\,d^7z_3\,d^5\bar z_4\,d^7z_5\,d^7z_6\,\frac{\delta^3 S}{\delta v(z_3)\delta v(z_2)\delta q(z_1)}\cdot\frac{\delta^3 S}{\delta v(z_6)\delta v(z_5)\delta\bar q(z_4)} \\ &\quad\times\Big(G_{+-}(z_1,z_4)G(z_2,z_5)G(z_3,z_6)+G_{+-}(z_1,z_4)G(z_2,z_6)G(z_3,z_5)\Big).\end{aligned} \tag{5.16}$$

Consider them separately.

Using the $\bar D^2$ operators in the vertex (5.12) it is possible to restore full superspace measure in some of the integrals in (5.15) and to perform these integrals using the delta-functions. Then, we get the following representation for (5.15)

$$\Gamma_{B_1} = -4\int d^7z_1\,d^7z_2\,G_{+-}(z_1,z_2)G_{-+}(z_1,z_2)G(z_1,z_2)\,. \tag{5.17}$$

This analytic expression is represented by the Feynman graph Γ_{B_1} in Fig. 2. Next, we use the

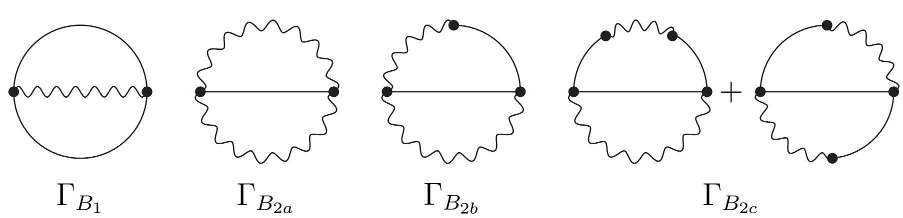

Figure 2: Two-loop Feynman graphs with cubic vertices.

derivatives D^2 and $\bar{D}^2$ in the propagator $G_{+-}(z_1,z_2)$ and integrate them by parts such that the identities (A.6), (A.7) and (A.9) can be used. It is easy to see that owing to these properties of the propagator $G(z,z')$ the contribution (5.17) vanishes on the considered background (2.5)

$$\Gamma_{B_1} = 0\,. \tag{5.18}$$

Let us consider now the part of effective action (5.16). Using the operators D^2 and $\bar{D}^2$ in the vertices (5.13) and (5.14) we restore full superspace measures and perform some of the integrals using the superspace delta-functions. Then, the contribution (5.16) can be represented as a sum of the following terms

$$\Gamma_{B_2} = \Gamma_{B_{2a}} + \Gamma_{B_{2b}} + \Gamma_{B_{2c}}\,, \tag{5.19}$$

where

$$\Gamma_{B_{2a}} = -16\int d^7z_1\,d^7z_2\,\bar{Q}Q\,G_{+-}(z_1,z_2)G(z_1,z_2)G(z_1,z_2)\,, \tag{5.20}$$

$$\Gamma_{B_{2b}} = 64\int d^7z_1\,d^7z_2\,d^7z_3\,\bar{Q}Q\,G_{+-}(z_2,z_3)G_{+-}(z_1,z_2)G(z_1,z_3)G(z_1,z_2)\,, \tag{5.21}$$

$$\begin{aligned}\Gamma_{B_{2c}} &= -32\int d^7z_1\,d^7z_2\,d^7z_3\,d^7z_4\,\bar{Q}Q\,G_{-+}(z_1,z_2)G_{+-}(z_3,z_4)G_{+-}(z_1,z_3)\\ &\quad\times\Big(G(z_1,z_3)G(z_2,z_4)+G(z_1,z_4)G(z_2,z_3)\Big)\,.\end{aligned} \tag{5.22}$$

These contributions to the effective action correspond to the Feynman graphs $\Gamma_{B_{2a}}$, $\Gamma_{B_{2b}}$ and $\Gamma_{B_{2c}}$ in Fig. 2. The details of the computations of these diagrams are given in Appendix A.2. Only the final results are written down here:

$$\begin{aligned}\Gamma_{B_{2a}} &= -\frac{4}{k^2}\int d^7z\;\bar{Q}Q\left(\frac{1}{\varepsilon}-\gamma-2\ln\frac{\bar{Q}Q}{k\mu}\right)+\frac{8\ln 2}{k^2}\int d^7z\,\bar{Q}Q\\ &\quad+\frac{4}{k^2\alpha^2}\int d^7z\,\bar{Q}(z)\Big(\frac{1}{\varepsilon}-\gamma\Big)Q(z)\,,\end{aligned} \tag{5.23a}$$

$$\begin{aligned}\Gamma_{B_{2b}} &= -\frac{8}{k^2\alpha^2}\int d^7z\,\bar{Q}(z)\Big(\frac{1}{\varepsilon}-\gamma\Big)Q(z)\\ &\quad-\frac{8}{k^2}\int d^7z\;\bar{Q}Q\left(\frac{1}{\varepsilon}-\gamma-2\ln\frac{\bar{Q}Q}{k\mu}\right)-\frac{4(1-2\ln 2)}{k^2}\int d^7z\,\bar{Q}Q\,,\end{aligned} \tag{5.23b}$$

$$\Gamma_{B_{2c}} = \frac{4}{k^2\alpha^2}\int d^7z\,\bar{Q}(z)\Big(\frac{1}{\varepsilon}-\gamma\Big)Q(z)\,. \tag{5.23c}$$

Here ε is the parameter of dimensional regularization, $d=3-2\varepsilon$, $\varepsilon\to 0$, and γ is the Euler constant. Let us collect the divergent and finite terms in (5.23) separately

$$\Gamma_{B_2} = \Gamma_{B_2,\mathrm{div}} + \Gamma_{B_2,\mathrm{fin}}\,, \tag{5.24}$$

$$\Gamma_{B_2,\mathrm{div}} = -\frac{12}{\varepsilon k^2}\int d^7z\,\bar{Q}Q\,, \tag{5.25}$$

$$\Gamma_{B_2,\mathrm{fin}} = \int d^7z\,\bar{Q}Q\left(\frac{16\ln 2-4+12\gamma}{k^2}+\frac{24}{k^2}\ln\frac{\bar{Q}Q}{k\mu}\right)\,. \tag{5.26}$$

In is important to note that all terms containing the gauge-fixing parameter α cancel each other out in (5.23) and the final result (5.24) is α-independent. This result is expected since we have taken the gauge-independent part of the effective action into account.

The divergent contribution to the effective action (5.25) has the structure of the classical action for the chiral superfield. It can be eliminated by adding the corresponding counterterm to the bare action (2.1). Further we concentrate only on the finite terms which contribute to the effective Kähler potential.

5.3 Two-loop effective Kähler potential

The sum of two-loop finite contributions to the effective action (5.8) and (5.26) can be written as

$$\Gamma^{(2)}_{\rm fin} = -\int d^7z\, K^{(2)}\,, \tag{5.27}$$

where $K^{(2)}$ is the two-loop quantum correction to the effective Kähler potential

$$K^{(2)} = 2\bar{Q}Q\left(\frac{4}{k^2} - \frac{8\ln 2}{k^2} - \frac{6}{k^2}\gamma - \frac{12}{k^2}\bar{Q}Q\,\ln\frac{\bar{Q}Q}{k\mu}\right). \tag{5.28}$$

Let us now consider the full effective Kähler potential which comprises the one-loop (4.17b) and two-loop (5.28) quantum contributions as well as the classical part

$$K_{\rm eff}(\bar{Q},Q) = 2\bar{Q}Q\left(1 - \frac{1}{k} + \frac{4}{k^2} - \frac{8\ln 2}{k^2} - \frac{6\gamma}{k^2} - \frac{12}{k^2}\,\ln\frac{\bar{Q}Q}{k\mu}\right). \tag{5.29}$$

Here μ is the normalization point which is usually fixed from the condition

$$\left.\frac{\partial^2 K_{\rm eff}(\bar{Q},Q)}{\partial\bar{Q}\partial Q}\right|_{Q=Q_0} = 2\,. \tag{5.30}$$

With such a normalization we get the final expression for the effective Kähler potential in the two-loop approximation

$$K_{\rm eff} = 2\,\bar{Q}Q - \frac{24}{k^2}\,\bar{Q}Q\left(\ln\frac{\bar{Q}\,Q}{\bar{Q}_0 Q_0} - 2\right). \tag{5.31}$$

The effective Kähler potential (5.31) deserves the following comments:

- The form of the effective Kähler potential is very similar to the one-loop effective Kähler potential in four-dimensional $\mathcal{N}=1$ supersymmetric gauge theories interacting with chiral matter which were studied in [2, 8–10]. In fact, this form is universal in the sense that it is dictated by logarithmic quantum divergences which appear in one loop in four dimensions and start from two loops in three-dimensional model under considerations.

- We did all the quantum computations keeping the gauge-fixing parameter α arbitrary, but found that the effective Kähler potential (5.31) is independent of α. This is a manifestation of the fact that we computed the low-energy effective action on the background (2.5) which solves not only the classical, but also effective quantum equations of motion in the one-loop approximations.

- Let us consider the model of $\mathcal{N}=2$ supersymmetric electrodynamics (2.1) which is known to be superconformal at the classical level. The effective Kähler potential explicitly breaks the superconformal invariance since it involves dimensional parameters Q_0 and $\bar{Q}_0$. Thus, the superconformal invariance is broken by two-loop quantum corrections.

6 Conclusions

In the present paper, we have computed the two-loop effective Kähler potential in the $\mathcal{N}=2$ SQED with Chern-Simons kinetic term for the gauge superfield. The result (5.31) resembles the four-dimensional one-loop effective Kähler potential [2, 8–10] since its form is stipulated by logarithmic quantum divergences.

The calculations have been done in the framework of the background field method with a one-parametric family of gauges. It was proven that the resultant effective Kähler potential is gauge independent. Also, we want to emphasize that we have used the "non-standard" change of variables in the functional integral to diagonalize the propagator. Such a procedure creates the new non-local interaction vertices in the supergraphs. We have shown that these new vertices do not lead to obstacles in computations. In the conclusions, let us discuss the possible future development of the obtained results.

The most obvious generalization is to consider non-Abelian $\mathcal{N}=2$ Chern-Simons matter theories. We expect that the form of the effective Kähler potential in these theories should be similar to (5.31), but many more quantum computations are required since one has to take into account more Feynman graphs in non-Abelian theories including, in particular, ghost field contributions. We will leave these issues for further studies.

It is possible to include more parameters in the classical action such as the masses of chiral matter superfields and the Yang-Mills gauge coupling. It is interesting to study how the effective Kähler potential depends on the values of all these parameters.

The Chern-Simons-matter theories with $\mathcal{N}>2$ supersymmetry are known to be UV-finite [25–27]. Hence, the effective Kähler potential in these models can receive only finite quantum corrections. For $\mathcal{N}=4$ supersymmetric models there is a non-renormalization theorem [37, 38] which forbids perturbative quantum corrections to the moduli space in the Higgs branch described by the effective Kähler potential. However, it is not clear whether this applies to $\mathcal{N}=3$ supersymmetric gauge-matter theories. Therefore, it would be interesting to study a structure of the effective Kähler potential in the $\mathcal{N}=3$ gauge theory.

Acknowledgments

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A Gauge superfield propagator

In this Appendix we consider some useful properties of the gauge superfield propagator introduced in sect. 3. Recall that, after gauge fixing and performing the non-local change of fields (3.7), the quadratic action for gauge superfields is defined by the operators (3.14) and (3.15). Using the identity

$$(\bar{D}^\alpha D_\alpha)^2 = 4\Box - \frac{1}{4}\{D^2, \bar{D}^2\} \tag{A.1}$$

the operator of quadratic fluctuations of the gauge superfield can be rewritten as

$$\begin{aligned} H + M - \Delta &= H + M - M\frac{\{D^2, \bar{D}^2\}}{16\Box} = H + \frac{M}{4\Box}(\bar{D}^\alpha D_\alpha)^2 \\ &= \frac{ik}{4\pi}\bar{D}^\alpha D_\alpha + \frac{M}{4\Box}(\bar{D}^\alpha D_\alpha)^2 + \frac{ik\alpha}{8\pi}(\bar{D}^2 + D^2)\,. \end{aligned} \tag{A.2}$$

Then, it is easy to check that the distribution

$$G(z,z') = \left[\frac{i\pi}{k}\frac{\bar{D}^\alpha D_\alpha}{\Box + \frac{4\pi^2 M^2}{k^2}} - \frac{\pi^2 M}{k^2}\frac{(\bar{D}^\alpha D_\alpha)^2}{\Box(\Box + \frac{4\pi^2 M^2}{k^2})} + \frac{i\pi}{2k\alpha}\frac{\bar{D}^2 + D^2}{\Box}\right]\delta^7(z - z') \tag{A.3}$$

solves the equation for the gauge superfield propagator

$$(H + M - \Delta)G(z,z') = -\delta^7(z - z')\,. \tag{A.4}$$

Using (A.1) the propagator (A.3) can be identically rewritten as

$$\begin{aligned} G(z,z') \;=\; &\left[-\frac{4\pi^2 M}{k^2}\frac{1}{\Box + \frac{4\pi^2 M^2}{k^2}} + \frac{i\pi}{k}\frac{\bar{D}^\alpha D_\alpha}{\Box + \frac{4\pi^2 M^2}{k^2}} + \frac{\pi^2 M}{4k^2}\frac{\{\bar{D}^2, D^2\}}{\Box(\Box + \frac{4\pi^2 M^2}{k^2})}\right. \\ &\left. + \frac{i\pi}{2k\alpha}\frac{\bar{D}^2 + D^2}{\Box}\right]\delta^7(z - z')\,. \end{aligned} \tag{A.5}$$

It is straightforward to check that (A.5) has the following useful properties

$$\bar{D}^2 G(z,z') = \frac{i\pi}{2k\alpha}\frac{\bar{D}^2 D^2}{\Box}\delta^7(z - z')\,, \tag{A.6}$$

$$\frac{D^2\bar{D}^2}{16}G(z,z') = \frac{i\pi}{2k\alpha}D^2\delta^7(z - z') = -\frac{2i\pi}{k\alpha}\,\delta_-(z,z')\,, \tag{A.7}$$

where $\delta_-(z,z')$ is the antichiral delta function.

In loop quantum computations, we need the expressions for the gauge superfield propagator

(A.5) and its derivatives at coincident Grassmann coordinates

$$G(z,z')\Big|_{\theta=\theta'} = \frac{8\pi^2 M}{k^2}\frac{1}{\Box(\Box+\frac{4\pi^2M^2}{k^2})}\delta^3(x-x')\,, \tag{A.8}$$

$$\bar{D}^2D^2G(z,z')\Big|_{\theta=\theta'} = 0\,, \tag{A.9}$$

$$\bar{D}^2G(z,z') = D^2G(z,z')\Big|_{\theta=\theta'} = \frac{8i\pi}{k\alpha}\frac{1}{\Box}\delta^3(x-x')\,, \tag{A.10}$$

$$\bar{D}^\alpha D_\alpha\, G(z,z')\Big|_{\theta=\theta'} = -\frac{8i\pi}{k}\frac{1}{\Box+\frac{4\pi^2M^2}{k^2}}\,\delta^3(x-x')\,, \tag{A.11}$$

$$D_\beta\bar{D}_\alpha G(z,z')\Big|_{\theta=\theta'} = \frac{4i\pi}{k}\frac{1}{\Box+\frac{4\pi^2M^2}{k^2}}\left[\varepsilon_{\alpha\beta}-\frac{2\pi M}{k}\frac{\partial_{\alpha\beta}}{\Box}\right]\delta^3(x-x')\,, \tag{A.12}$$

$$\frac{D^2\bar{D}^2}{16}D_\beta\bar{D}_\alpha G(z,z')\Big|_{\theta=\theta'} = \frac{4i\pi}{k}\frac{\Box}{\Box+\frac{4\pi^2M^2}{k^2}}\left[\varepsilon_{\alpha\beta}-\frac{2\pi M}{k}\frac{\partial_{\alpha\beta}}{\Box}\right]\delta^3(x-x')\,. \tag{A.13}$$

B Some details of computations of two-loop diagrams

In this Appendix we collect some details of quantum computations of two-loop Feynman graphs which were considered in sect. 5. We start by considering the graphs in Fig. 1.

B.1 Diagram Γ_{A_2}

The Feynman graph $\Gamma_{A_{2a}}$ in Fig. 1 contains two gauge superfield propagators which meet at one quartic vertex. We will use this propagator in the form (A.5). Using the superspace delta-function which is present in this propagator we integrate over one set of Grassmann variables θ'

$$\begin{aligned}\Gamma_{A_{2a}} &= 4\int d^7z\,d^7z'\,\delta^7(z-z')\bar{Q}Q\,G(z,z')\,G(z,z')\\ &= \frac{4\cdot 64\pi^4}{k^4}\int d^7z\,d^3x'\,\delta^3(x-x')\bar{Q}Q\,M^2\\ &\quad\times\frac{1}{\Box(\Box+\frac{4\pi^2M^2}{k^2})}\delta^3(x-x')\,\frac{1}{\Box(\Box+\frac{4\pi^2M^2}{k^2})}\delta^3(x-x')\,.\end{aligned} \tag{B.1}$$

Here we used the identity (A.9). For bosonic delta-functions in (B.1) we perform the Fourier transform and compute the resulting momentum integral using (C.1)

$$\begin{aligned}\Gamma_{A_{2a}} &= \frac{4\cdot 64\pi^4}{k^4}\int d^7z\,\bar{Q}Q\,M^2\left(\int\frac{d^3p}{(2\pi)^3}\frac{1}{p^2(p^2-\frac{4\pi^2M^2}{k^2})}\right)^2\\ &= -\frac{4}{k^2}\int d^7z\,\bar{Q}Q\,.\end{aligned} \tag{B.2}$$

This contribution to the effective action is finite and has the form of classical action for a free chiral superfield Q.

B.2 Diagrams Γ_{B_2}

Consider now the details of quantum computations of Feynman graphs given in Fig. 2.

B.2.1 Diagram $\Gamma_{B_{2a}}$

The diagram $\Gamma_{B_{2a}}$ in Fig. 2 contains two gauge superfield propagators and one (anti)chiral propagator which meet at two cubic vertices. For the gauge superfield propagator $G(z, z')$ we will use the representation (A.5) while the (anti)chiral propagator is given by (3.10)

$$
\begin{aligned}
\Gamma_{B_{2a}} &= -16 \int d^7 z_1\, d^7 z_2\, \bar{Q}Q\, G_{+-}(z_1, z_2) G(z_1, z_2) G(z_1, z_2) \qquad \text{(B.3)} \\
&= -16 \int d^7 z_1\, d^7 z_2\, \bar{Q}Q\, G_{+-}(z_1, z_2)\, G(z_1, z_2) \\
&\times \Bigg[-\frac{4\pi^2 M}{k^2} \frac{1}{\Box + \frac{4\pi^2 M^2}{k^2}} + \frac{i\pi}{k} \frac{\bar{D}^\alpha D_\alpha}{\Box + \frac{4\pi^2 M^2}{k^2}} + \frac{\pi^2 M}{4k^2} \frac{\{\bar{D}^2, D^2\}}{\Box(\Box + \frac{4\pi^2 M^2}{k^2})} \\
&\quad + \frac{i\pi}{2k\alpha} \frac{\bar{D}^2 + D^2}{\Box} \Bigg] \delta^7(z_1 - z_2)\,. \qquad \text{(B.4)}
\end{aligned}
$$

We rewrite the last integrals as a sum of four terms

$$
\begin{aligned}
\Gamma_{B_{2a}} \;=\; & \frac{64\pi^2}{k^2} \int d^7 z_1 d^7 z_2\, \bar{Q}QM\, G_{+-}(z_1, z_2)\, G(z_1, z_2) \frac{1}{\Box + \frac{4\pi^2 M^2}{k^2}} \delta^7(z_1 - z_2) \\
& - \frac{16 i\pi}{k} \int d^7 z_1 d^7 z_2\, \bar{Q}Q\, G_{+-}(z_1, z_2)\, G(z_1, z_2) \frac{\bar{D}^\alpha D_\alpha}{\Box + \frac{4\pi^2 M^2}{k^2}} \delta^7(z_1 - z_2) \\
& - \frac{4\pi^2}{k^2} \int d^7 z_1 d^7 z_2\, \bar{Q}QM\, G_{+-}(z_1, z_2)\, G(z_1, z_2) \frac{\{\bar{D}^2, D^2\}}{\Box(\Box + \frac{4\pi^2 M^2}{k^2})} \delta^7(z_1 - z_2) \\
& - \frac{8 i\pi}{k\alpha} \int d^7 z_1 d^7 z_2\, \bar{Q}Q\, G_{+-}(z_1, z_2)\, G(z_1, z_2) \frac{\bar{D}^2 + D^2}{\Box} \delta^7(z_1 - z_2)\,. \qquad \text{(B.5)}
\end{aligned}
$$

In first line in (B.5) we use the explicit form of the full superspace delta-function $\delta^7(z_1 - z_2) = \delta^4(\theta_1 - \theta_2)\delta^3(x_1 - x_2)$ and integrate over θ_2 using (A.8). In the second line in (B.5) we integrate by parts the derivatives of the $\bar{D}^\alpha D_\alpha$ operator and then integrate over θ_2 using (A.11). In the third line in (B.5) we integrate by parts the covariant spinor derivatives contained in the Green function $G_{+-}(z_1, z_2)$ and integrate over θ_2 using (A.8). After this, the terms in the first and third lines cancel against each other.

The term in the second line of (B.5) can be rewritten as

$$
\Gamma_{B_{2a}} = -\frac{128\pi^2}{k^2} \int d^7 z_1\, d^3 x_2\, \bar{Q}Q \frac{1}{\Box} \delta^3(x_1 - x_2) \frac{1}{\Box + \frac{4\pi^2 M^2}{k^2}} \delta^3(x_1 - x_2) \frac{1}{\Box + \frac{4\pi^2 M^2}{k^2}} \delta^3(x_1 - x_2)\,. \qquad \text{(B.6)}
$$

Passing to the momentum representation we integrate over space-time variable x_2 and calculate

the momentum integrals using (C.2) and (C.3)

$$
\begin{aligned}
\Gamma_{B_{2a}} &= \frac{128\pi^2}{k^2}\int d^7z\,\bar{Q}Q\int\frac{d^3p\,d^3q}{(2\pi)^6}\frac{1}{(p+q)^2(p^2-\frac{4\pi^2M^2}{k^2})(q^2-\frac{4\pi^2M^2}{k^2})}\\
&= -\frac{4}{k^2}\int d^7z\,\bar{Q}Q\left(\frac{1}{\varepsilon}-\gamma-2\ln\frac{\bar{Q}Q}{k\mu}\right)+\frac{8\ln 2}{k^2}\int d^7z\,\bar{Q}Q\,. \quad (\varepsilon\to 0) \qquad \text{(B.7)}
\end{aligned}
$$

Let us consider the term in the last line in (B.5). We integrate by parts the covariant spinor derivatives which are contained in the chiral superfield propagator (3.10) keeping in mind the identity (A.10)

$$
\begin{aligned}
&-\frac{8i\pi}{k\alpha}\int d^7z_1d^7z_2\,\bar{Q}(z_1)Q(z_2)\,G_{+-}(z_1,z_2)\,G(z_1,z_2)\frac{\bar{D}^2+D^2}{\Box}\delta^7(z_1-z_2)\\
=&\ \frac{128\pi^2}{k^2\alpha^2}\int d^7z_1d^3x_2\,\bar{Q}(x_1,\theta_1)Q(x_2,\theta_1)\frac{1}{\Box}\delta^3(x_1-x_2)\frac{1}{\Box}\delta^3(x_1-x_2)\frac{1}{\Box}\delta^3(x_1-x_2)\,.
\end{aligned}
$$

Here we also integrated over θ_2. Next, we pass to the momentum representation and integrate over x_1 and x_2

$$
-\frac{128\pi^2}{k^2\alpha^2}\int d^4\theta\int\frac{d^3l}{(2\pi)^3}\bar{Q}(l,\theta)Q(-l,\theta)\int\frac{d^3pd^3q}{(2\pi)^6}\frac{1}{p^2q^2(p+q+l)^2}\,. \qquad \text{(B.8)}
$$

This momentum integral can be evaluated using (C.4) after the change of integration variable $p\to p-l$

$$
\begin{aligned}
&-\frac{128\pi^2}{k^2\alpha^2}\int d^4\theta\int\frac{d^3l}{(2\pi)^3}\bar{Q}(l,\theta)Q(-l,\theta)\int\frac{d^3pd^3q}{(2\pi)^6}\frac{1}{(p-l)^2q^2(p+q)^2}\\
&=\frac{4}{k^2\alpha^2}\int d^4\theta\int\frac{d^3l}{(2\pi)^3}\bar{Q}(l,\theta)\Big(\frac{1}{\varepsilon}-\gamma-\ln(-l^2)\Big)Q(-l,\theta)\,, \qquad \text{(B.9)}
\end{aligned}
$$

In the coordinate representation this expression reads

$$
\frac{4}{k^2\alpha^2}\int d^7z\,\bar{Q}(z)\Big(\frac{1}{\varepsilon}-\gamma-\ln\Box\Big)Q(z)\,. \quad (\varepsilon\to 0) \qquad \text{(B.10)}
$$

Note that the last term in (B.10) contains the operator $\ln\Box$ which can be discarded for the constant superfield background (2.5) which we use to compute the effective Kähler potential.

Finally, we combine the two non-trivial contributions (B.7) and (B.10) and get the following result for the quantum contributions corresponding to the Feynman graph $\Gamma_{B_{2a}}$

$$
\begin{aligned}
\Gamma_{B_{2a}} &= -\frac{4}{k^2}\int d^7z\,\bar{Q}Q\left(\frac{1}{\varepsilon}-\gamma-2\ln\frac{\bar{Q}Q}{k\mu}\right)+\frac{8\ln 2}{k^2}\int d^7z\,\bar{Q}Q\\
&\quad+\frac{4}{k^2\alpha^2}\int d^7z\,\bar{Q}(z)\Big(\frac{1}{\varepsilon}-\gamma\Big)Q(z)\,. \qquad \text{(B.11)}
\end{aligned}
$$

Note that it contains divergent quantum contributions which appear as the pole $\frac{1}{\varepsilon}$. These divergent contributions should be removed by adding the corresponding counterterm to the classical action after computing all divergent two-loop diagrams.

B.2.2 Diagram $\Gamma_{B_{2b}}$

The diagram Γ_{2b} in Fig. 2 corresponds to the following analytic expression

$$\Gamma_{B_{2b}} = 64 \int d^7z_1\, d^7z_2\, d^7z_3\, \bar{Q}Q\, G_{+-}(z_2,z_3)G_{+-}(z_1,z_2)G(z_1,z_2)G(z_1,z_3)\,. \qquad \text{(B.12)}$$

Integrating by parts the covariant spinor derivatives which are present in the (anti)chiral Green function $G_{+-}(z_1,z_2)$ we collect them on the product $G(z_1,z_2)G(z_1,z_3)$ and then apply the following identity

$$\begin{aligned} D_1^2\bar{D}_1^2 G(z_1,z_2)G(z_1,z_3)|_{\theta_2=\theta_1} &= \bar{D}^2G(z_1,z_2)D^2G(z_1,z_3) + D^2G(z_1,z_2)\bar{D}^2G(z_1,z_3) \qquad \text{(B.13)}\\ &\quad -2D^\beta\bar{D}^\alpha G(z_1,z_2)D_\beta\bar{D}_\alpha G(z_1,z_3) + G(z_1,z_2)D^2\bar{D}^2G(z_1,z_3)\,. \end{aligned}$$

Here we omit all terms which are equal to zero after the integration over θ_2. After that we integrate by parts the covariant spinor derivatives contained in $G_{+-}(z_2,z_3)$ and get

$$\begin{aligned} \Gamma_{B_{2b}} &= 4\int d^7z_1\, d^7z_2\, d^7z_3\, \bar{Q}Q\, \frac{1}{\Box}\delta^7(z_3-z_2)\frac{1}{\Box}\delta^7(z_1-z_2)\\ &\quad \times\frac{\bar{D}_3^2D_3^2}{16}\Big(\bar{D}^2G(z_1,z_2)D^2G(z_1,z_3) + D^2G(z_1,z_2)\bar{D}^2G(z_1,z_3)\\ &\quad -2D^\beta\bar{D}^\alpha G(z_1,z_2)D_\beta\bar{D}_\alpha G(z_1,z_3) + G(z_1,z_2)D^2\bar{D}^2G(z_1,z_3)\Big)\\ &= -\frac{(16\pi)^2}{k^2\alpha^2}\int d^7z_1\, d^3x_2\, \bar{Q}(x_1,\theta_1)Q(x_2,\theta_1)\frac{1}{\Box}\delta^3(x_1-x_2)\frac{1}{\Box}\delta^3(x_1-x_2)\frac{1}{\Box}\delta^3(x_1-x_2)\\ &\quad -\frac{(16\pi)^2}{k^2}\int d^7z_1\, d^3x_2\, d^3x_3\, \bar{Q}Q\frac{1}{\Box}\delta^3(x_3-x_2)\frac{1}{\Box}\delta^3(x_1-x_2)\\ &\quad \times\frac{1}{\Box+\frac{4\pi^2M^2}{k^2}}\delta^3(x_1-x_2)\frac{\Box}{\Box+\frac{4\pi^2M^2}{k^2}}\delta^3(x_1-x_3)\\ &\quad +\frac{2(4\pi)^4}{k^4}\int d^7z_1\, d^3x_2\, d^3x_3\, \bar{Q}Q\, M^2\frac{1}{\Box}\delta^3(x_3-x_2)\frac{1}{\Box}\delta^3(x_1-x_2)\\ &\quad \times\frac{\partial^m}{\Box(\Box+\frac{4\pi^2M^2}{k^2})}\delta^3(x_1-x_2)\frac{\partial_m}{\Box+\frac{4\pi^2M^2}{k^2}}\delta^3(x_1-x_3) \end{aligned} \qquad \text{(B.14)}$$

Here we have done the integrals over Grassmann variables θ_3 and θ_2 and used the properties of the gauge superfield propagator (A.6), (A.7) and (A.9)–(A.13). Next, we pass to the momentum representation in (B.14)

$$\begin{aligned} \Gamma_{B_{2b}} &= \frac{256\pi^2}{k^2\alpha^2}\int d^4\theta\int\frac{d^3l}{(2\pi)^3}\bar{Q}(l,\theta)Q(-l,\theta)\int\frac{d^3pd^3q}{(2\pi)^6}\frac{1}{(p-l)^2q^2(p+q)^2}\\ &+ \frac{256\pi^2}{k^2}\int d^7z\, \bar{Q}Q\int\frac{d^3p\, d^3q}{(2\pi)^6}\frac{1}{(p+q)^2(p^2-\frac{4\pi^2M^2}{k^2})(q^2-\frac{4\pi^2M^2}{k^2})}\\ &+ \frac{256\pi^4}{k^4}\int d^7z\, \bar{Q}Q\, M^2\int\frac{d^3p\, d^3q}{(2\pi)^6}\frac{2p\cdot q}{p^2(p^2-\frac{4\pi^2M^2}{k^2})(p+q)^2q^2(q^2-\frac{4\pi^2M^2}{k^2})} \end{aligned} \qquad \text{(B.15)}$$

and compute the momentum integrals using (C.3), (C.4) and (C.5)

$$
\begin{aligned}
\Gamma_{B_{2b}} &= -\frac{8}{k^2\alpha^2}\int d^7z\,\bar{Q}(z)\Big(\frac{1}{\varepsilon}-\gamma\Big)Q(z) \\
&\quad -\frac{8}{k^2}\int d^7z\,\bar{Q}Q\left(\frac{1}{\varepsilon}-\gamma-2\ln\frac{\bar{Q}Q}{k\mu}\right)-\frac{4(1-2\ln 2)}{k^2}\int d^7z\,\bar{Q}Q\,. \qquad \text{(B.16)}
\end{aligned}
$$

In this expression we omitted the terms containing the operator $\ln\Box$ since they do not contribute to the effective Kähler potential.

B.2.3 Diagram $\Gamma_{B_{2c}}$

The sum of two diagrams Γ_{2c} in Fig. 2 corresponds to the following analytic expression

$$
\begin{aligned}
\Gamma_{B_{2c}} &= -32\int d^7z_1\,d^7z_2\,d^7z_3\,d^7z_4\,\bar{Q}Q\,G_{-+}(z_1,z_2)G_{+-}(z_3,z_4)G_{+-}(z_1,z_3) \\
&\quad \times\left[G(z_1,z_3)G(z_2,z_4)+G(z_1,z_4)G(z_2,z_3)\right]\,. \qquad \text{(B.17)}
\end{aligned}
$$

Let us consider the contributions from the first term in the brackets in (B.17). We integrate by parts the covariant spinor derivatives contained in $G_{+-}(z_2,z_1)$ and $G_{-+}(z_4,z_3)$ and apply the identity (A.7)

$$
\begin{aligned}
&-32\int d^7z_1\,d^7z_2\,d^7z_3\,d^7z_4\,\bar{Q}Q \\
&\qquad\times G_{+-}(z_1,z_3)G(z_1,z_3)\frac{1}{\Box}\delta^7(z_2-z_1)\frac{1}{\Box}\delta^7(z_4-z_3)\frac{\bar{D}_4^2D_4^2}{16}\frac{D_2^2\bar{D}_2^2}{16}G(z_2,z_4) \\
&=\frac{64i\pi}{k\alpha}\int d^7z_1\,d^7z_2\,d^7z_3\,d^7z_4\,\bar{Q}Q \\
&\qquad\times G_{+-}(z_1,z_3)G(z_1,z_3)\frac{1}{\Box}\delta^7(z_2-z_1)\frac{1}{\Box}\delta^7(z_4-z_3)\frac{\bar{D}_4^2D_4^2}{16}\delta_-(z_2,z_4)=0\,. \qquad \text{(B.18)}
\end{aligned}
$$

This expression vanishes since there is the chiral delta-function $\delta_-(z_2,z_4)$ integrated over the full superspace.

The contribution from the second term in the brackets in (B.17) can be calculated in a similar way

$$
\begin{aligned}
\Gamma_{B_{2c}} &= -32\int d^7z_1\,d^7z_2\,d^7z_3\,d^7z_4\,\bar{Q}Q\,G_{-+}(z_1,z_2)G_{+-}(z_3,z_4)G_{+-}(z_1,z_3)G(z_1,z_4)G(z_2,z_3) \\
&= -32\int d^7z_1\,d^7z_2\,d^7z_3\,d^7z_4\,\bar{Q}Q\,\frac{1}{\Box}\delta^7(z_2-z_1)\frac{1}{\Box}\delta^7(z_4-z_3)\,G_{+-}(z_1,z_3) \\
&\qquad\times\frac{D_2^2\bar{D}_2^2}{16}G(z_2,z_3)\frac{\bar{D}_4^2D_4^2}{16}G(z_1,z_4)\,. \qquad \text{(B.19)}
\end{aligned}
$$

Here we integrated by parts the covariant spinor derivatives contained in $G_{-+}(z_1,z_2)$ and $G_{+-}(z_3,z_4)$. Then we do the integration over θ_2 and apply the identity (A.7)

$$
\begin{aligned}
\Gamma_{B_{2c}} &= -\frac{16i\pi}{k\alpha}\int d^7z_1\,d^3x_2\,d^7z_3\,d^7z_4\,\bar{Q}Q\,\frac{1}{\Box}\delta^3(x_2-x_1)\frac{1}{\Box}\delta^7(z_4-z_3)\frac{1}{\Box}\delta^7(z_1-z_3) \\
&\qquad\times\frac{D_1^2\bar{D}_1^2}{16}\left(D^2(x_2,\theta_1)\delta^4(\theta_1-\theta_3)\delta^3(x_2-x_3)\;\frac{\bar{D}_4^2D_4^2}{16}G(z_1,z_4)\right)\,. \qquad \text{(B.20)}
\end{aligned}
$$

The non-zero contribution arises only when the operator D_1^2 acts on $G(z_1, z_4)$. After integration over θ_4 and θ_3 we obtain two bosonic delta-functions $\delta^3(x_2 - x_3)$ and $\delta^3(x_1 - x_4)$ which allow us to do the integration over x_3 and x_4 as well

$$\begin{aligned}
\Gamma_{B_{2c}} &= \frac{128\pi^2}{k^2\alpha^2} \int d^7 z_1\, d^3 x_2\, \bar{Q}Q\, \frac{1}{\Box}\delta^3(x_2 - x_1)\frac{1}{\Box}\delta^3(x_1 - x_2)\frac{1}{\Box}\delta^3(x_1 - x_2) \\
&= \frac{4}{k^2\alpha^2} \int d^7 z\, \bar{Q}(z) \left(\frac{1}{\varepsilon} - \gamma\right) Q(z)\,.
\end{aligned} \tag{B.21}$$

Here we applied the formula (C.3) to compute the corresponding momentum integral.

C Momentum integrals

In this Appendix we give the list of momentum integrals which appear in one- and two-loop Feynman graphs considered in the present paper. Some of these integrals can be found in the textbook [39]:

$$J(m) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{(p^2)(p^2 - m^2)} = -\frac{i}{4\pi\,|m|}\,, \tag{C.1}$$

$$I(p,m) = \int \frac{d^3q}{(2\pi)^3} \frac{1}{(p+q)^2(q^2 - m^2)} = \frac{1}{8\pi} \int_0^1 \frac{dx}{\sqrt{1-x}\,(p^2x - m^2)^{1/2}} \tag{C.2}$$

$$I_1 = \int \frac{d^3p}{(2\pi)^3} \frac{I(p,m)}{p^2 - m^2} = -\frac{1}{32\pi^2}\frac{\Gamma(\varepsilon)}{m^{2\varepsilon}} + \frac{\ln 2}{16\pi^2}\,, \tag{C.3}$$

$$I_2 = \int \frac{d^3p}{(2\pi)^3} \frac{I(p,0)}{(p-l)^2} = -\frac{1}{32\pi^2}\frac{\Gamma(\varepsilon)}{(-l^2)^{\varepsilon}}\,, \tag{C.4}$$

$$I_3 = \int \frac{d^3p\, d^3q}{(2\pi)^6} \frac{2p\cdot q}{p^2(p^2 - m^2)(p+q)^2 q^2(q^2 - m^2)} = -\frac{1 + 2\ln 2}{16\pi^2\, m^2}\,. \tag{C.5}$$

Here ε is the parameter of dimensional regularization, $d = 3 - 2\varepsilon$, $\varepsilon \to 0$. The divergent parts of these integrals can be singled out by the standard series expansion of the Γ-function

$$\frac{\Gamma(\epsilon)}{m^{2\epsilon}} = \frac{1}{\epsilon} - \gamma - \ln m^2 + O(\epsilon)\,, \tag{C.6}$$

where $\gamma = 0,577...$ is the Euler constant.

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